RESOURCES FOR TEACHING: SUPPORTING A MEXICAN TEACHER'S LEARNING

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Abstract
We analyze the role played by a teaching resource in effectively supporting a professional development collaboration with Irene, a Mexican public-school teacher. The resource is an instructional sequence on fractions that was developed through a series of design experiments in Mexican classrooms. As a result of this collaboration, Irene modified significantly her instructional practices. We discuss how the instructional sequence contributed to Irene's renewed view of a mathematics classroom by providing her with guidance that was explicit, specific, and achievable.

Introduction

Mathematics teaching is not the same around the world. National educational systems have their unique histories and are organized differently. Mathematics teachers work under different institutional conditions, receive dissimilar opportunities for their professional development, and engage with students whose cultural, social, and educational backgrounds often vary substantially.

We explore the case of a Mexican public-school teacher, Irene, who agreed to collaborate with us in a dual design experiment (Gravemeijer and van Eerde 2009), aimed at supporting both her professional development as a mathematics teacher, and her fifth-grade students' understanding of measurement and fractions. As a result of this collaboration, Irene modified significantly her instructional practices. In addition, she successfully supported her fifth-grade students in making sense of important mathematical ideas. This was especially significant given that very few Mexican children seem to get a fair opportunity to understand the targeted ideas as they go through compulsory education.

In this paper, we analyze the role played by an instructional resource in effectively supporting the professional development collaboration with Irene (cf. Pepin 2018). The resource in question is not a printed textbook, but an instructional sequence on fractions as measures. We elucidate the difference and discuss how the instructional sequence contributed to Irene's renewed view of a mathematics classroom by providing her with guidance that was *explicit*, *specific,* and *achievable*.

After we situate our study, we introduce the instructional sequence, the key principles that guided its design, and anticipations for its use. We then overview the dual design experiment, focusing on its professional development component and outcomes resulting from Irene's work in her classroom. Finally, we introduce our analysis of supports that facilitated Irene's transition to more ambitious and equitable instructional practice (Jackson, Gibbons, and Sharpe 2017).

Background

To situate our contribution, we refer to the state of mathematical learning in Mexico, particularly with regard to pupils living in harsh social and economic circumstance. For more than fifteen years, national and international assessments have shown a rather disturbing image. For instance, in a recent assessment conducted by the National Institute of Educational Evaluation (Instituto Nacional para la Evaluación de la Educación 2015b), 60.5% of sixth graders perform in mathematics below

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the "basic" level. The data also shows that the great majority of those low performing pupils are the children of families living in poverty.

Studies in which instructional resources are developed are not often situated in severely underprivileged classrooms, but teachers in these classrooms require support. Moreover, such classrooms are not unique to Mexico. Data form PISA (Organisation for Economic Co-operation and Development 2013) suggests that just within Latin America, where the ICMT took place, Argentina, Brazil, Chile, Colombia, Costa Rica, Peru, and Uruguay face similar challenges.

In Mexico, more than a decade of efforts at the public policy level included changing the curriculum, developing new official textbooks, implementing high-stakes tests, and establishing programs of special rewards for good teachers. Yet, these efforts did not result in a clear indication of improvement of students' learning or performance. In this context, we hope to systematically explore the role that instructional sequences that are a product of careful design and experimentation in classrooms, can play in supporting teachers who work with low performing students. We situate our contribution within the space of teachers' resources (Trouche and Fan 2018) and discuss implications of our analysis for conceptualization of educative curriculum materials (Davis and Krajcik 2005) designed for these teachers.

THE INSTRUCTIONAL SEQUENCE

We developed the instructional sequence on fractions as measures (Cortina, Visnovska, and Zúñiga 2014, 2015) to respond to concerns about limitations in the mathematical competence that most Mexican students achieve in their formal education. The results from both national and international assessments indicate that very few Mexican students develop the necessary mathematical understandings to mathematize and solve problems that involve continuous magnitudes, or require the use of rational numbers or multiplicative reasoning. For instance, in PISA 2012 (Organisation for Economic Co-operation and Development 2013) only 17% of Mexican sixteen-year-olds achieved proficiency levels (Levels 3 and above) that involve ability to handle percentages, fractions, and decimal numbers, and to work with proportional relationships. In terms of quantitative literacy (Steen 2001), this suggests that the majority of Mexican children leave schooling able to soundly deal with only a few types of quantitative situations – those limited to natural numbers, additive relations, and discreet quantities.

The instructional sequence was developed through a series of classroom design experiments in Mexican classrooms, following Cobb and colleagues' methodological guidelines (Gravemeijer & Cobb 2006). Framed by the design theory of Realistic Mathematics Education (RME; Gravemeijer 1994), the sequence entails classroom activities that are experientially real for students, can guide them to reinvent mathematics by bringing in their everyday experiences, and provide the students with opportunities to create their own mathematical models (Cobb 2003).

In addition, the sequence was developed under the assumption that teachers necessarily adjust the instructional resources they use to the actual circumstances that they encounter in their classrooms. Hence, the sequence was not conceived as an instructional asset that could influence students' learning directly. Instead, it was developed as a resource for supporting teachers in pursuing a fruitful instructional agenda (Cobb, Zhao, and Visnovska 2008).

As an instructional resource, the instructional sequence is not a printed collection of lessons, problems, and exercises that a teacher and her students can follow. Instead, it outlines a progression of students' learning goals, along with the rationale for this progression, which includes the means of supporting the students' learning at each step. Following RME, the purpose of this rationale is to support the teacher in making informed instructional decisions as she adjusts the instructional activities to the contingencies she encounters in her classroom. This would include decisions such as when to start pursuing a new learning goal and how to do it.

The instructional sequence on fractions as measures is intended to support a classroom community in reinventing length measurement, including the complexities of measuring the reminders of units.

The students are first expected to develop the need to *standardize the unit* that serves as a reference when measuring lengths. They then confront the problem of measuring the lengths that the reference unit does not cover exactly (i.e., remainders). Unit fractions are then introduced as a means of producing measurement subunits—smaller than the reference unit—in a systematic way. Within the sequence, it is expected that common fractions will come to be construed by students as quantities that express the number of times that a specific subunit was iterated when measuring a certain length. For instance, it is expected that the fraction 7/5 will come to be construed as seven iterations of the length of a subunit *1/5 as long as the reference unit*. Finally, this way of construing fractions is expected to allow students to gauge the lengths that fractions account for as either being shorter than (e.g., 5/7), as long as (e.g., 7/7), or longer than (e.g., 7/5) the reference unit. The rationale for this sequence includes notions that typically fall within teacher's pedagogical repertoire, such as how classroom activities within the sequence can be organized to be most productive and why, and what types of classroom discourse made it possible for students to progress in past trials.

Working in four different Mexican schools, we have documented how the instructional sequence can be a powerful resource in supporting low-achieving and disenfranchised children, in developing relatively sophisticated understandings of fractions as measures (Cortina, Visnovska & Zúñiga 2014). These understandings include the inverse order relation amongst unit fractions (Tzur 2007), and fractions as numbers that may account for the size of quantities bigger than one unit (Norton and Hackenberg 2010). We have also recognized that the effective use of the instructional sequence in Mexican classrooms entails great teaching challenges. It requires making instructional decisions based on students' reasoning, as well as supporting pupils to participate in ways that are new to them and often uncommon in their regular classrooms. They are expected to actively listen to what others say, ask questions, express non-understanding, and articulate and communicate their own thinking.

Collaborating with Irene

In several ways, Irene can be regarded as a typical Mexican elementary-school teacher. Almost all of her students were the children of low-income families, and most of them were low achievers in mathematics. Irene graduated from Mexico City's Normal School, and became the first member of her family to be a teacher and to obtain a college-level degree. She was amongst the 68% of Mexican elementary school-teachers who are graduates of public normal schools, with no postgraduate education (Instituto Nacional para la Evaluación de la Educación 2015a).

Throughout her sixteen-year teaching career, Irene had worked in public elementary schools located in the hilly western suburbs of Mexico City. As many other Mexican teachers, she rarely collaborated with her colleagues on issues directly related to instruction. As it is also typical, school authorities held her accountable mostly on matters concerning administrative issues, such as completing paperwork correctly and on time. She received no support for her teaching, but also a very little oversight of how she taught on a daily basis. Even though official regulations are rather restrictive in the Mexican educational system, Irene had relatively high autonomy in deciding what to teach, when to teach it, and how to teach it in her classrooms.

In other ways, Irene can be regarded as an atypical Mexican teacher. She was amongst the few who held two teaching positions, one in a school with a morning shift (8am to 1pm) and the other in an afternoon shift (2 to 7pm). She was also a teacher who was unsatisfied with her students' mathematical achievement, felt responsible for it, and believed that by furthering her education she could improve on what her students can learn.

In 2014, Irene obtained a paid leave of absence in her morning-shift school position, to enroll in a master's program at the National Pedagogical University (NPU). She continued with her teaching in the afternoons. Our collaboration with her commenced at the beginning of her master's studies, with the second author as her academic adviser.

The NPU master's program has a strong professional development orientation. It is expected that enrolled students' research projects will further their capacity as mathematics teachers. The dual design experiment thus presented an ideal framework for supporting and researching Irene's learning. It allowed us to collect data on Irene's adoption of a new resource, while she conducted the classroom design experiment on fractions, for her master's project, in her fifth-grade afternoon classroom. Irene obtained ethical clearance from the university and consent from the students, parents, and school administrators, to collect students' work and videotape all the classroom sessions.

Methodology

The data collection for the dual design experiment primarily consisted of two design research logs. Irene's *planning and teaching log* included elaborated lesson plans, in which she specified the learning goals for each upcoming classroom session, and the activities she planned to use. After each classroom session, Irene annotated her lesson plan, reflecting on classroom events. Irene also video-recorded all the teaching sessions, and collected copies of her students' work.

The second author produced a *research log*, which included design conjectures and notes related to both students' and Irene's learning. First, the log documented the second author and Irene's conversations during weekly debriefing and planning meetings aimed at understanding students' learning progress. Second, this log documented weekly to bi-weekly debriefing sessions between the two authors, which focused on Irene's teaching and planning, and on the ways in which her work was supported.

In the retrospective analysis of the data, we relied on an adaptation of the constant comparative method described by Cobb and Whitenack (1996) that involves testing and revising tentative conjectures while working through the data chronologically. As we analyzed new teaching episodes, we compared these with conjectured themes and categories. This process resulted in a set of the theoretical assertions that remained grounded in the data. For present purposes, we focused on the key episodes, which highlighted features of the instructional sequence on which Irene relied as she supported the learning of diverse learners in her classroom.

Irene's Classroom Design Experiment Overview

In six one-hour weekly sessions with the second author, Irene first became acquainted with the instructional sequence, including how it was developed and used in prior classroom design experiments. She worked through all the instructional activities as a student, deepened her understanding of measurement and fractions, and got acquainted with the different aspects of instructional practices that place student mathematical reasoning at the center of decision-making in the classroom. She also became familiar with the classroom design experiment methodology and the importance of documenting her rationales for instructional decisions made in the process.

Irene started to trial the instructional sequence in her fifth-grade classroom about four months into the school year. The results of her students' initial assessment suggested that the great majority of them had made little progress in making sense of fractions as numbers that express quantities. For instance, all but four of her 20 students would not correctly and consistently recognize which of two fractions (e.g., 1/4 and 1/2) represented the bigger amount. Moreover, only two of her fifth-grade students seemed to recognize that improper fractions account for quantities that are bigger than one.

Irene worked with the instructional sequence during 18 weekly sessions, each lasting about 35 minutes. After each classroom session, she met with the second author, for one hour, to analyze the learning that took place, and plan for the upcoming session. Irene described the different ways in which students participated and how they reasoned during the instructional activities. When planning for the upcoming session, particular attention was placed on supporting the participation of the students that struggled the most.

The results of a final written assessment suggested that, similar to other classrooms in which the instructional sequence had been used, Irene's students developed relatively sophisticated understandings of fractions as measures. They could all easily and correctly compare unit fractions.

In addition, all but two students could readily gauge the size of a common fraction as being smaller than, as big as, or bigger than one, and could use this knowledge to make accurate estimates of where to place a fraction on a number line (Visnovska & Cortina 2017).

Trialing the instructional sequence had a profound impact on Irene's teaching. We became aware of this by noticing that in the final three instructional sessions, the students in Irene's class made explanations that were clearer and more articulate than those made by students in any of the other groups in which we had worked with the instructional sequence. When we asked Irene about her thoughts on why this had happened, she responded that for several months now, she had been asking students to express non-understanding and to communicate their thoughts, regardless of what she was teaching. Whole-class conversations had become a common aspect of her everyday teaching, in all subject areas. She mentioned that she now felt uncomfortable teaching without knowing what her students were understanding, particularly those students who struggled the most.

Many factors can indeed be recognized as being critical in influencing Irene's teaching, including her dissatisfaction with the results she was obtaining, her willingness to learn, and the intense professional development support she received during her classroom design experiment. Nonetheless, as professional development facilitators, we recognize that the instructional sequence became a particularly powerful means of supporting Irene's learning. In the following section, we identify and analyze some of the key roles that the sequence played.

The instructional sequence as a resource for teaching

To understand the supports the sequence provided, we need to discuss ways in which Irene's instruction changed. At the beginning of the collaboration, Irene was mostly concerned about delivering instruction appropriately. She would decide what to do and how based on the contents specified in the program of studies, and on the lessons included in the official textbook and in other teaching resources that she regarded as valuable. She believed that if she taught "well," her students would learn. Unfortunately, many of her students made little progress in mastering the content she taught. At the time of her entry to the masters' program, the pending issue for her was to learn what kind of instruction would be more effective, and how can it be implemented properly.

During the classroom design experiment, Irene's focus on proper enactment was replaced by her emerging need to understand how her students were thinking. We illustrate this through an episode from the very beginning of her classroom design experiment, where Irene set out activities for her students to measure with body parts. She followed the instructional sequence where the first learning goal for students was to become aware of the shortcomings of using body parts as units of measure. Noticing these shortcomings was an important step for students to come to see the introduction of a standard (informal) measurement unit, *the stick*, as a reasonable innovation.

The instructional activities Irene used in her classroom were introduced in a conversation about how people measured before there were rulers and tape measures. Based on the prior experiences in using the instructional sequence, it was expected that students would come to value practicality of measuring with body parts as they measured objects in the classroom with their hands. In addition, it was expected that some pupils would become aware of disadvantages in measuring with body parts. Specifically, Irene expected that some students would find it problematic that the class sometimes recorded different measures for the same measured length.

For the most part, the session unfolded as Irene expected. Students had considered using body parts, and had eagerly engaged in measuring and recording the measures they made with their hands. However, in the whole-class conversation, students had only said positive things about measuring with body parts. Irene first interpreted this classroom session in a way consistent with the orientation she typically followed in her teaching. She considered that she did not properly implement the activities, since the students had not come up with the expected contributions. Even though the learning goal had not been achieved, she planned to proceed to activities in which students would now start using a standard unit of measure that she would introduce, a wooden stick.

In the debriefing session, the second author shared with Irene how the instructional sequence aims to structure students' experience of instructional activities as being *coherent*. He explained how this aim guides us to only introduce a new tool such as the stick to students once they can see it as a solution to a problem that they already identified. Otherwise, from students' perspective, an introduction of a new tool would break the storyline, and the tool would be seen as an object important to the teacher but with no particular value for students. Irene considered the classroom situation from the students' perspective, and agreed to co-design and trial in her classroom additional instructional activities with the goal of helping her students recognize the shortcomings of measuring with body parts.

When Irene and the second author met again, she reported that the instructional activities had worked fairly well. In one of them, she had told the students how a window had once broken in their school. The principal in the morning shift had measured its width, obtaining five hands. Then the principal of the afternoon shift had measured its width and obtained six hands. They were puzzled about which of them had made a mistake when measuring.

Irene related that most of the students had recognized that both principals could have measured correctly, and that measuring with hands might thus not always be a good idea. Surprisingly to the second author, Irene did not yet want to proceed to the following mathematical goal (this would involve Irene asking how we could improve on measuring with hands, and later introducing a new measurement tool, the stick). Instead, she suspected that for several of her pupils the shortcomings of using non-standardized units of measure were unclear and she wanted to create additional scenarios in which the problem could be discussed. She now wanted to make sure that *all* her students had reached the first learning goal before pursuing a new one.

From this point on, a significant shift in Irene's orientation as a mathematics teacher was noticeable. She was now making instructional decisions based on what she considered her students were understanding and what they needed to learn next. She no longer based these on a plan about what had to be taught. For this shift to happen, it was critical that the instructional sequence she was trialing had an explicit rationale that specified a progression of clearly formulated and specific learning goals, and that these goals were achievable in her classroom. We now unpack each of these supports in turn.

Learning goals: Explicit rationale

For more than twenty years, the Ministry of Education has been providing Mexican teachers with "sequences of problem situations" that are expected "to arouse students' interest, and invite children to reason mathematically, find ways of solving problems, and formulate arguments that validate their discoveries" (Secretaría de Educación Pública 2011, 67). Although teachers have been provided with guides that include many recommendations about how to best enact the sequences of problem situations in their classroom, and even to complement them, little information is usually offered to the teachers about the rationale that guided the design. For the most part, this knowledge is implicit in the provided sequences, as it seems to be the case with many carefully crafted instructional resources. According to Remillard (2018), accessibility to design rationales remains problematic even in resources that are accompanied by teaching guides, such as ones produced as part of NSF-funded textbook series in the US.

The rationale for the instructional sequence that Irene trialed with her students was explicitly formulated during the process of initial design and subsequent modifications in a number of classrooms, and consists of conjectures about the collective mathematical development of the classroom community, and how this development can be supported. Availability of this rationale made it possible for the second author to discuss with Irene her role in supporting each of the learning goals that students were expected to sequentially achieve, and the resources on which she may draw in doing so, including the ideas that students generate. Had the instructional sequence been limited to a set of instructional activities, manipulatives, worksheets, and other tools, about

which we knew merely that they worked in other classrooms, it would have been very difficult to guide Irene in responding to the unexpected student ideas that emerged in her classroom.

Learning goals: Specific and understandable

In addition to availability of the rationale for the instructional sequence, it was also important that the student learning goals were specific. Efforts for reforming mathematics education, at least in Mexico, have often included comprehensive recommendations for students' learning, such as to help pupils build their own knowledge and skills with sense and meaning (Secretaría de Educación Pública 2011). Although valuable at some level, goals that are this broad might not be of much use in making specific instructional decisions – including how to know whether to move to the next learning activity.

During the debriefing after the initial instructional session, it was apparent that Irene understood the first learning goal, and that it was specific enough for her, so much so that she clearly recognized that it had not been reached. Had the goals been formulated in a way that was not easy for her to assess – because she found them to be vague or confusing – it would have been much harder for her to focus her instructional agenda on her students' learning.

The specificity of the learning goals positioned Irene as a teacher fully capable of reflecting accurately on her progress. Her advisor, in turn, became a collaborator in a shared problem-solving situation, rather than a person exerting judgment or control over her work. Irene identified that her students did not accomplish the learning goal. Discussing pedagogical issues related to the sequence rationale helped her re-evaluate why this was problematic, and realize that the problem would not be addressed by moving to the next type of measurement activity. Similarly to students realizing that different-sized hands were problematic, Irene's refined awareness of the problem enabled her to look for a more adequate solution. The importance of sequentially achieving each of the learning goals then became more meaningful to her.

Learning goals: Achievable in the classroom

Once Irene recognized the importance of reaching the first learning goal, it was crucial that she also regarded it as achievable by all the students in her classroom. This seemed to have happened during the second instructional session, when she noticed how many of her students, including ones she did not expect to do so, started to realize that measuring with non-standardized units might not always be a good idea. At that point, she seems to have realized that, with more support, everyone in the classroom could come to the same conclusion. This realization allowed her to put aside her concern for what needed to be taught, and start making instructional decisions based on her observations and conjectures about what were her students understanding. Importantly, observing how they accomplished the first learning goal seems to have provided Irene with confidence in the instructional sequence, and a resolve that the ensuing goals could also be sequentially reached.

It is important to point out that the achievement of Irene's students on the first learning goal was not a fortuitous occurrence, by any means. As we mentioned before, the sequence she was trialing was a product of careful design and experimentation in classrooms with students very similar to hers. As designers, we were not only fairly confident that the expected goal was achievable for Irene's students within several classroom sessions, we could also draw on the sequence rationale to advise Irene about various means of support that the sequence provided for her use (e.g., ways of creating additional problem scenarios).

Concluding remarks

The instructional sequence used in our professional development collaboration with Irene is a rather sophisticated resource for teaching, in many ways different from traditional textbooks. As noted throughout this paper, its effective use entails great challenges. It requires a kind of teaching that is complex, not typical, and involves substantial learning (Maaß & Artigue 2013).

It might thus seem unsound to consider the instructional sequence on fractions as measures as a worthy resource in an educational system like the Mexican one, where ambitious and equitable

instructional practices are not typical. However, this kind of instruction can also be regarded as necessary for significant improvement in Mexican students' mathematical understanding to take place, particularly for those that are living in harsh social and economic circumstance. From this perspective, our professional development collaboration with Irene can be seen as an illustrative case of a specific kind of resource that can successfully support Mexican teachers' transition towards instructional practices that better support their students' understanding of worthy mathematical ideas. We discussed several features of the instructional sequence on fractions as measures and how these facilitated teacher learning in relation to the instructional design rationale for the sequence.

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